LITERATURE CITED

1. R. G. Shreffler and R. H. Christian, "Boundary disturbance in high-explosive shock tubes," J. Appl. Phys., 25, No. 3 (1954).
2. M. A. Tsikulin and E. G. Popov, Radiative Properties of Shock Waves in Gases [in Russian], Nauka, Moscow (1977).
3. S. D. Savrov and I. M. Ageev, "A laboratory explosive shock tube," Teplofiz. Vys. Temp., No. 6 (1980).
4. K. E. Gubkin, "Propagation of blast waves," in: Mechanics in the USSR over the Past 500 Years [in Russian], Vol. 2, Nauka, Moscow (1970).

APPROXIMATE CALCULATION OF STEADY-STATE SHOCK WAVE PARAMETERS IN POROUS COMPRESSIBLE MATERIALS

L. G. Gvozdeva and Yu. M. Faresov

UDC 532.593:532.529

Formulation of the Problem. Analysis of Calculation Model. Elastic polyurethane foams based on simple and complex polyethers [1] are among the class of materials characterized by high porosity (the total pore volume may reach 98% of the total specimen volume) and low skeleton elasticity values. The characteristic size of polyurethane foam pores depends on the brand and method of preparation, ranging from fractions to units of mm. We will consider a porous material with sufficiently large linear specimen dimensions (hundreds of mm and more) as a mixture of solid particles and a gas, which may be justified as follows. The skeleton elasticity of the majority of polyurethane foam brands is markedly less than the value of atmospheric pressure under normal conditions (thus, for the widely used type PPU-ÉM-1 with apparent density of 25-45 kg/m^3 the skeleton stress at 40% deformation comprises $(0.4\text{-}1.0)\cdot 10^4$ Pa [1]).

This indicates that it is possible to neglect the skeleton elasticity of such materials in comparison to the elasticity of the gas contained in the pores at an initial gas pressure $p^0 \sim 10^5$ Pa and model the real porous material by a liquid with gas bubbles or a homogeneous gaseous suspension of solid noninteracting microparticles.

The dynamics of propagation of acoustic or shock waves in such media were analyzed in [2-6] by representing the two-phase medium as a homogeneous mixture. As was noted in [6], the homogeneous mixture is the simplest possible model of a two-phase medium in which all effects related to the discrete nature of the structure are neglected, and only the volume fraction of the solid phase which effects the compressibility of the medium as a whole is considered. We will assume the solid phase to be incompressible while the gas within the pores is ideal. For complete thermodynamic description of a homogeneous mixture it is necessary to specify an exact or approximate law of interphase heat exchange for the concrete type of porous material. This is because within the small radius of the pores interphase thermal equilibrium in the two-phase porous medium can be established over a time less than the characteristic duration of propagating acoustical perturbations, so that it is permissible to consider the medium to be in thermodynamic equilibrium and even isothermal (in view of the considerable specific heat of the solid phase). Such an approximation was used in [2-6]; however, in [3] an approximate estimate was also made of the cooling time of an isolated gas bubble of radius r in a liquid initially adiabatically compressed from a pressure p_1 to p_2 (T_0 = 21°C):

$$t_1 \approx 43r^2(p_2/p_1)^{0.05},\ t_2 \approx 8600r^2(p_3/p_1)^{1/3}, \qquad (1)$$

where t_1 and t_2 are the times required for cooling of the bubble by 10 and 90%, respectively; r is measured in m. Applying this estimate to elastic polyurethane and taking $r \sim 10^{-3}$ m (the characteristic value of the pore radius) we find in the acoustical approximation $t_1 \simeq 5\cdot 10^{-5}$ sec, $t_2 \simeq 10^{-2}$ sec. When the rate of propagation of hydrodynamic perturbations in the medium exceeds the value r/t_1, it is appropriate to assume heat exchange responsible and consider an "adiabatic" model of flow of the two-phase medium. At $r \sim 10^{-3}$ m and $t_1 \simeq 5\cdot 10^{-5}$ sec

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 120-125, January-February, 1986. Original article submitted November 21, 1984.

0021-8944/86/2701-0107$12.50

the ratio $r/t_1 \simeq 50$ m/sec. Experimental measurements of the speed of sound in polyurethane foam (see below) indicate that the inequality $a > 50$ m/sec is almost always satisfied (at least at gas pressures in the pores $p^0 \geqslant 10^5$ Pa). The speed of sound in a homogeneous mixture modeling real porous materials with a quasi-inelastic skeleton is defined by the expression [7]

$$a^2 = \frac{1}{[\alpha\rho^0 + (1-\alpha)\rho^1]\,[\alpha/\rho^0(a^0)^2 + (1-\alpha)/\rho^1(a^1)^2]}, \tag{2}$$

where ρ^0, ρ^1 are the densities of the gaseous and solid phases, respectively; α is the volume gas content; a^0, a^1 are the speeds of sound in the gas and solid phase. At $\alpha > 10^{-3}$ Eq. (2) simplifies significantly:

$$a^2 \simeq \frac{(a^0)^2}{\alpha(1-\alpha)\rho^1/\rho^0}. \tag{3}$$

According to Eq. (3), at $\alpha = 0.5$ the speed of sound has its minimum value. With adiabatic compression of the gas in the pores $(a^0)^2 = \gamma p^0/\rho^0$ (p^0 is the gas pressure), whence we have

$$a^2 \simeq \gamma p/(\alpha(1-\alpha)\rho^1). \tag{4}$$

The density of the homogeneous mixture is defined by the expression

$$\rho = \alpha\rho^0 + (1-\alpha)\rho^1. \tag{5}$$

When the gas pressure in the pores $p^0 \leqslant 10^5$ Pa, Eq. (4) with consideration of Eq. (5) can be rewritten in a form more convenient for calculations,

$$a^2 \simeq \gamma p^0/\alpha\rho. \tag{6}$$

Use of Eqs. (4), (6) for calculation of the function $a(p^0)$ in a porous medium is especially convenient when the medium has an open cell structure, since in that case $\alpha = \alpha_0$ and is independent of gas pressure. Thus, for the widely used open cell foam PPU-ÉM-1 at $\alpha = 0.98$, $\rho = 25$ kg/m^3 and $p^0 = 10^5$ Pa the quantity a calculated by Eq. (6) comprises $\sim$77 m/sec, which is significantly less than the speed of sound in air under normal conditions.

We will consider relationships on the front of a steady-state shock wave, propagating in a two-phase porous medium represented as a homogeneous mixture. As before, we will neglect skeleton elasticity of the porous material in comparison to the elasticity of the gas in the pores, which permits significant simplification of the system of equations of conservation of mass, momentum, and energy on the pressure discontinuity. Further simplification of the problem involves neglect of gas filtration through the permeable structure of the porous material, which is not obviously justifiable, but requires experimental validation. Analysis of the experimental results of [8, 9] does indicate that for relatively strong shock waves in elastic polyurethane foam with small cells at a pressure head at the front $\Delta p \geqslant (10\text{-}15)\cdot 10^5$ Pa and initial pressure of 10^5 Pa the gas filtration effect is insignificant, showing that it is acceptable in the present case to neglect filtration. If porous plastic materials with a closed cell structure are used filtration is absent automatically.

We will assume further that interphase heat exchange on the density discontinuity may be neglected, i.e., we will consider an adiabatic model of flow of the two-phase medium. In fact, with a characteristic polyurethane foam pore radius $r \sim 10^{-3}$ m and a pressure ratio on the front $p_2/p_1 \gtrsim 1$ it follows from Eq. (1) that $t_1 \gtrsim 5\cdot 10^{-5}$ sec. Since this value is comparable to experimentally measured values of the duration of the shock front in elastic polyurethane foam $\sim 10^{-4}$ sec [8, 9], it can be assumed that the process of gas compression in the pores upon transition through the discontinuity is practically adiabatic. We will now use the subscript 1 to denote medium parameters ahead of the discontinuity and the subscript 2 for parameters behind the discontinuity. We will transform to a coordinate system fixed to the discontinuity. With consideration of the assumptions made, the system of equations of conservation of mass, momentum, and energy will not differ from the corresponding system of equations for the case of an ideal gas:

$$\begin{gathered}\rho_1 v_1 = \rho_2 v_2, \quad \rho_1 v_1^2 - \rho_2 v_2^2 = p_2 - p_1, \\ \frac{p_1}{\rho_1} + U_1 + \frac{v_1^2}{2} = \frac{p_2}{\rho_2} + U_2 + \frac{v_2^2}{2},\end{gathered} \tag{7}$$

where ρ_i, v_i, p_i, U_i ($i = 1, 2$) are the density, mass velocity, pressure, and specific internal energy of the medium in the given region of the flow. It is obvious that in the absence

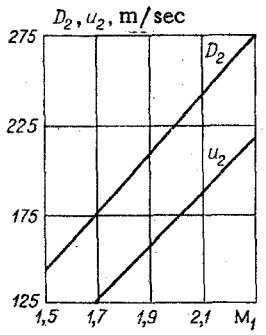

Fig. 1

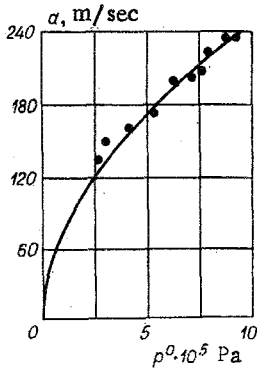

Fig. 2

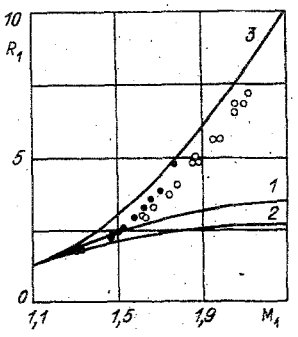

Fig. 3

of interphase heat exchange, when the internal energy of the medium is taken equal to the internal energy of the ideal gas in the free pore volume, the expression $U\rho = U^0\rho^0\alpha$ is valid (here and below the superscript 0 denotes parameters of the gaseous media), whence

$$U = \frac{1}{\gamma - 1}\frac{p\alpha}{\rho}. \tag{8}$$

The values of the porosity coefficients α_1, α_2 on both sides of the discontinuity are related by an expression easily obtained using the assumption of incompressibility of the solid phase of the medium:

$$\alpha_2 = 1 - (1 - \alpha_1)\rho_2/\rho_1. \tag{9}$$

Substituting in Eq. (7) the quantities U_1, U_2 calculated with the aid of Eq. (9) we have the desired relationship between the densities and pressures on the discontinuity:

$$\frac{\rho_2}{\rho_1} = \frac{p_2/p_1(\gamma+1) + (\gamma-1)}{p_2/p_1(\gamma+1-2\alpha_1) + (2\alpha_1+\gamma-1)}. \tag{10}$$

This expression is an analog of the Hugoniot adiabat for a two-phase medium, and as can easily be proved, for $\alpha_1 = 1$ (a pure gas) it takes on the classical form [10]. In a high intensity shock wave $p_2/p_1 \to \infty$, so from Eq. (10) we have the value of the limiting compression of the medium

$$\left(\frac{\rho_2}{\rho_1}\right)_\infty = \frac{\gamma+1}{\gamma+1-2\alpha_1}.$$

For example, for $\gamma = 1.4$ and $\alpha_1 = 0.95$ the limiting compression $(\rho_2/\rho_1)_\infty = 4.8$, i.e., markedly less than in the case of a pure gas, where $(\rho_2/\rho_1)_\infty = 6$. With the aid of shock adiabat equation (10) it is simple to obtain an expression in explicit form for the shock wave velocity in the medium D_2 and the mass velocity behind the front u_2 (laboratory coordinate system):

$$D_2 = \sqrt{\frac{p_2(\gamma+1) + p_1(\gamma-1)}{2\alpha_1\rho_1}}, \quad u_2 = (p_2 - p_1)\sqrt{\frac{2\alpha_1}{\rho_1[p_2(\gamma+1) + p_1(\gamma-1)]}}. \tag{11}$$

Using Eq. (11) and the approximate expression for the frozen speed of sound, Eq. (6), one can express the Mach number of the shock wave as a function of the gasdynamic parameters

$$M_2 = \frac{D_2}{a} \simeq \sqrt{\frac{p_2(\gamma+1) + p_1(\gamma-1)}{2p_1\gamma}}.$$

We will consider the process of reflection of a steady-state air shock wave from a wall covered by a layer of elastic polyurethane foam. To calculate the parameters of the shock wave formed in the porous medium it will be convenient to use a graphoanalytical variant of the method of decay of an arbitrary discontinuity of state parameters in a gas [11], which in addition to the pressure in the shock wave p_2 allows determination of the velocity of the gas–porous material boundary u_2 for a given intensity of the air shock wave. Calculations of the relative reduction in pressure on the phase boundary $K(M_1) = p_2/p_{20}$ (where p_{20} is the pressure of normal reflection of the incident shock wave with M_1 in air from a rigid wall) indicate that with increase in M_1 in the interval 1.5-2.3 for initial conditions $p_1 = 10^5$ Pa, $\rho_1 = 33$ kg/m^3, $\alpha_1 = 0.975$ the coefficient $K(M_1)$ decreases monotonically from 0.76 to 0.66. Figure 1 shows curves of $D_2(M_1)$, $u_2(M_1)$ calculated with the aid of Eq. (11) for these initial conditions.

Considering the equation of momentum balance for the incident and reflected shock waves in the medium and applying Eq. (10) it can be shown that the analytical expression for the reflection coefficient $R_0 = p_3/p_2$ (the subscript 3 denotes the parameters of the reflected shock wave) is identical to the classical expression [10] for the case where the medium is an ideal gas.

Denoting the initial air pressure by p_0^0, we write the final expression for the coefficient of pressure increase on the wall $R_1 = p_3/p_{20}$:

$$R_1(M_1) = K(M_1)\frac{(3\gamma-1)K(M_1)p_{20}(M_1)/p_0^0-(\gamma-1)}{(\gamma-1)K(M_1)p_{20}(M_1)/p_0^0+(\gamma+1)}. \quad (12)$$

Experimental Studies. To verify the computation model described, the above experiments were performed during which pressure pulses on a wall were measured, together with the speed of sound in an elastic open cell type PPU-ÉM-1 polyurethane foam for various initial conditions. The experimental apparatus consisted of a rectangular shock tube with diaphragm, with cross section 0.1 × 0.1 m. The length of the high-pressure chamber was 1.5 m, and the total length of low-pressure section and measurement section was 8 m. Helium and air were used for the driver and working gases. For the experiments measuring the speed of sound in the polyurethane the high pressure chamber was filled with the material to be studied, after which air was pumped into this chamber until the copper diaphragm separating it from the evacuated shock tube channel ruptured.

Piezoelectric pressure sensors mounted in the side walls of the high pressure chamber (measurement base 0.2 m) and a dual trace oscilloscope were used to record pressure profiles in the rarefaction wave which propagated through the material after rupture of the diaphragm. Measurement of the time shift between the normalized pressure profiles allowed determination of the speed of sound in the medium for a given initial gas pressure. The oscilloscope was triggered by an additional piezosensor located in the high pressure chamber near the diaphragm which drove a trigger pulse shaper. Preliminary experiments confirmed that because of the open cell structure of the PPU-ÉM-1 foam with sufficiently slow gas supply (rate of pressure increase $\cong 5\cdot10^3$ Pa/sec) there was no irreversible deformation of the specimen which would hinder comparison of experimental and calculated data. The specimen was maintained at a gas pressure close to that required for rupture of the diaphragm for no less than 30 sec in each experiment. The accuracy to which the speed of sound was measured over the given base was determined mainly by the accuracy with which the pressure profiles were processed as well as the quality of the piezosensors, and comprised ∿20%. Results of the speed of sound measurements in PPU-ÉM-1 polyurethane foam are shown in Fig. 2, which also shows the calculated curve $a(p^0)$ constructed with Eq. (4) for the parameters of the material used in the experiments ($\alpha_1 = 0.98$, $\rho^1 \simeq 1.2\cdot10^3$ kg/m^3). Analysis of Fig. 2 reveals satisfactory quantitative correspondence between the calculated and experimental a values.

Experimental measurements of the pressure increase coefficient on the wall beneath a layer of polyurethane foam were performed with the shock tube described above. The experimental setup and some results were described in [9]. Plane layers of the material studied (PPU-ÉM-1, $\rho \simeq 25$ kg/m^3) ∿0.13 m thick were placed on the end face of the tube, completely covering the cross section. The initial pressure in the low pressure chamber was 10^5 Pa. Helium and nitrogen were used as driver gases, with the former producing a shock wave in the experimental section characterized by a falling pressure profile. This effect, not found when nitrogen was used, is related to the fact that the rarefaction wave reflected from the face of the high-pressure chamber was able to overtake the shock wave and form a nonsteady state pressure profile in the latter. The Mach number of the incident shock wave was varied over the range 1.3-2.3.

Figure 3 shows results of measurement of the coefficient R_1 for various values of M_1 (light and dark circles correspond to the two series of experiments using helium and nitrogen), as well as calculated curves 1 and 2, constructed with Eq. (14) at $\gamma = 1.4$ (adiabatic model) and $\gamma = 1.0$ (equilibrium model), respectively. Satisfactory quantitative agreement of the calculated and experimental R_1 values occurs only for $M_1 \lesssim 1.6$; at $M_1 > 1.6$ the experimental points lie above the calculated curves. As an example, Fig. 3 shows calculated curve 3, constructed with Eq. (14) for $\gamma = 1.0$ and $R_1(M_1)$ calculated for an adiabatic flow model. It can be shown that curve 3 defines the maximum possible values of $R_1(M_1)$ for the given initial conditions. It is evident that with increase in M_1 the experimental values of R_1 are described approximately by curve 3, which indirectly indicates the appearance of interphase heat exchange effects in the shock wave in the porous material.

Without presenting a detailed treatment of the processes leading to formation of a shock wave front in the porous compressible material, we will note that the front duration should significantly exceed the values characteristic of shock waves in gases. In the general case the shock wave front thickness in a medium with fine structure corresponds to some characteristic dimensions of the medium d, in the present case, the pore diameter. Assuming for simplicity that the front is formed at a distance $l \sim 10d$, we obtain for $d \sim 10^{-3}$ m and a shock wave velocity in the medium $D_2 \sim 2 \cdot 10^2$ m/sec a front duration $\tau \sim l/D_2 = 5 \cdot 10^{-5}$ sec, which agrees fairly well with the experimental data of [8, 9]. The permeability of the skeleton of real porous materials may lead to additional increase in the front duration.

The application range of the calculation method described is limited to relatively high initial gas pressures in the compressible porous medium ($p_0 \gtrsim 10^4$ Pa for elastic polyurethane). At lower gas pressures it is necessary to consider the elastic force of the skeleton material and modify Eq. (8) for the internal energy of the porous medium correspondingly.

LITERATURE CITED

1. V. M. Kataev, V. A. Popov, and B. I. Sazhin (eds.), Handbook of Plastics [in Russian], Vol. 2, Khimiya, Moscow (1975).
2. I. J. Campbell and A. S. Pitcher, "Shock waves in a liquid containing gas bubbles," Proc. Roy. Ser. A, 243, No. 1235 (1958).
3. B. R. Parkin, F. R. Gilmore, and G. L. Broad, "Shock waves in water with air bubbles," in: Underwater and Underground Explosions [Russian translation], Mir, Moscow (1977).
4. G. Rudinger, "The effect of a finite volume occupied by particles on the dynamics of gas and particle mixing," RTK, 3, No. 7 (1965).
5. A. Mallock, "The damping of sound by frothy liquids," Proc. Roy. Soc., A84, No. 391 (1910).
6. L. Van Weingarden, "One-dimensional liquid flows with gas bubbles," in: Rheology of Suspensions [Russian translation], Mir, Moscow (1975).
7. A. Vud, Sound Waves and Their Applications [in Russian], Gostekhteorizdat, Moscow–Leningrad (1934).
8. B. E. Gel'fand, S. A. Gubin, et al., "Study of features of pressure wave propagation and reflection in a porous medium," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1975).
9. L. G. Gvozdeva and Yu. M. Faresov, "Interaction of a shock wave with a wall coated by porous compressible material," Pis'ma Zh. Tekh. Fiz., No. 19 (1984).
10. K. P. Stanyukovich, Nonsteady State Motion of a Continuous Medium [in Russian], Nauka, Moscow (1971).
11. T. V. Bazhenova and L. G. Gvozdeva, Nonsteady State Shock Wave Interactions [in Russian], Nauka, Moscow (1977).